

Large N Cosmology

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Abstract

We motivate inflationary scenarios with many scalar fields, and give a complete formulation of adiabatic and entropy perturbations. We find that if the potential is very flat, or if the theory has a $SO(N)$ symmetry, the calculation of the fluctuation spectrum can be carried out in terms of merely two variables without any further assumption. We do not have to assume slow roll or $SO(N)$ invariance for the background fields. We give some examples to show that, even if the slow roll assumption holds, the spectrum of fluctuations can be quite different from the case when there is a single inflaton.

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1 Introduction

As phenomenological scenarios of the early Universe, inflationary models constructed with a single inflaton are popular for their simplicity and consistency with existing observations. However, for a coherent understanding of the Universe, cosmological models should be embedded in fundamental particle theories. As we will argue now, models with many inflatons may be more natural than those with a single inflaton.

It is often natural to have a large number of fields in various models for the early Universe. In Kaluza-Klein theories or string theory, it is very common to have a large number of moduli fields. For some [1], the enormous moduli space is the most important feature of string theory. For example, in the brane world scenario, there is a scalar field associated with each extra dimension, corresponding to brane fluctuations in that direction. In addition, for n D-branes on top of each other, each of these scalar fields is promoted to a $n \times n$ matrix. There are $6 \cdot 5^2 = 150$ independent real scalar fields for 5 D3-branes in 10 dimensions, for instance. As another example, for the tachyon model of inflation [2], it was pointed out that it is hard to be consistent with observations [3], unless there is a large number of coincident branes [4]. In fact, merely the 10 dimensional metric already gives 6 vectors and 15 scalars in addition to the metric in 4 dimensions. Recently, the possibility of inflation in pure gravity due to compactifications on time-varying hyperbolic spaces has attracted some attention and is being explored [5]. The scalars coming from the higher dimensional metric play the role of inflatons in these models. Based on the above examples, it may be quite unnatural to have a model of inflation with a single inflaton field. In fact, even the standard model has 3 or 4 real components from the Higgs doublet; and there are generically a large number of scalar fields in any grand unified theory (GUT).¹

Another motivation for multiple inflatons comes from the observation that in order to have a scale invariant spectrum of density perturbations, we need the slow roll condition $\left| \frac{\dot{H}}{H^2} \right| \ll 1$ to be satisfied. This implies that we need the potential energy of the inflatons to dominate over the kinetic energy. A naive scaling of the potential and kinetic energy terms with the number N of inflatons indicates that this condition is automatically satisfied in the large N limit, as we will show in Sec.2.²

Inflation driven by multiple scalar fields is not a new idea. One of the examples

¹In $SU(5)$ GUT, there is a Higgs field in the adjoint representation with 24 components, and another in the fundamental representation with 5 components. There are a lot more Higgs fields for $SO(10)$ GUT's.

²However, this scaling does not apply to assisted inflation.

which has been considered is the so-called “assisted inflation” scenario [6, 7], where exponential potentials are analyzed, and it was found that inflation is more easily achieved when there are a large number of scalar fields. As another example, density perturbations of many scalar fields which interact with one another only through gravity (but can have arbitrary self-interacting potentials) were studied in [8]. General analytic formulae for density perturbations in the multi-field inflation scenario have already been derived [9, 10, 11].

However, as we will see, it is in general a lot more complicated to deal with more than one field. In the case of a single inflaton, the slow roll condition implies that the slope of the potential is small. But this may not be true when there are more scalar fields, as will be explained in Sec.4. Furthermore, for N scalar fields, one needs to solve N coupled 2nd order differential equations for the background fields, and N additional coupled equations for the perturbations of each field. For a large number N this can be a formidable task even with a computer.

Despite the complexity of generic multiple inflaton models, the system can be greatly simplified in special cases. In [6] it was found that the background evolution only depends on a single variable if the scalars are free fields (and the inflation is driven by the cosmological constant). Later it was shown [12] that, for exponential potential, many scalar fields can be assembled into an adiabatic and an entropy mode by making a suitable rotation in field space, and the same result was obtained for polynomial potentials $\sum_i V(\phi_i)$ in [13], where each field has a potential of the same functional form V . In this paper, we extend the analysis to the non-trivial case when the inflatons observe an $SO(N)$ global symmetry. We find that only a single variable is needed to describe the evolution of the background geometry, and, more remarkably, only two equations for two independent variables are needed for the purpose of determining adiabatic and entropy perturbations. Note that, although we assume $SO(N)$ symmetry for the interaction, the background field configuration does not have to be $SO(N)$ invariant.

More specifically, we will focus on the case when the scalar fields are in the fundamental representation of the $SO(N)$ symmetry. We expect that the same technique can be applied to more general cases with different $SO(N)$ representations, or when the symmetry group is different. In those cases the number of variables needed for a complete description of the perturbations may be larger than two. Our experience with the simplest case will be valuable for those generalizations.

Examples of scalar fields in the fundamental representation are not hard to find. The Higgs doublet in the standard model has four real components, which transform as

a fundamental representation under an $SO(4)$ symmetry, for which the Higgs potential is invariant. The Higgs potentials in grand unified theories do not observe any $SO(N)$ symmetry in general. But if we consider the weak field approximation, and keep only the quadratic terms in the potential, $SO(N)$ symmetry often emerges. For the fundamental and adjoint representations of $SU(5)$, for instance, the quadratic terms in the Higgs potential in $SU(5)$ GUT, $\text{Tr}(H^\dagger H)$ and $\text{Tr}(\Sigma^2)$, have $SO(10)$ and $SO(24)$ symmetries, respectively. For the brane-world scenario, there are scalar fields in the fundamental representation of $SO(N)$ if there are extra N (flat) dimensions.

Imposing a symmetry on the model not only simplifies the formulation and makes calculations accessible, it also helps to render the model consistent with observational constraints on the magnitude of the entropy perturbation mode. Without the symmetry, each matter field gains different amounts of fluctuations according to its coupling to each inflaton. The predicted CMB perturbations will obtain contributions not only from the adiabatic mode of the metric perturbations, but also from the perturbations of each matter field that interacts with photons. Observational data shows that the adiabatic mode dominates, and thus we have to avoid excessive isocurvature perturbations. However, if the coupling of each matter field to the inflatons is $SO(N)$ invariant, only $SO(N)$ invariant operators can contribute to CMB fluctuations. Hence, we argue that inflationary models with multiple inflatons and with a large symmetry group are the most promising models to study. Similar conclusion was also reached in [14].

This paper is organized as follows. After reviewing the general formulation for multiple scalar fields coupled to gravity, we give the evolution equations for the background in Sec.2, and formulae for adiabatic and entropy perturbations in Sec.3. We studied perturbations for the following two circumstances: (1) theories with very flat potentials (Sec.5), (2) theories with $SO(N)$ symmetry (Sec.6). In the first case, if we in addition to the condition on the potential assume the slow roll condition, we obtain the same scale invariant spectrum of metric perturbations as the case of a single inflaton. This result strengthens the case for the robustness of the connection between slow-roll inflation and having a scale invariant spectrum of fluctuations.

In the second case, in which the fields have a $SO(N)$ symmetry, we find that even if the background configuration is arbitrary, we still only need two variables to describe the adiabatic and entropy fluctuations. This is true because the coupling of the metric to the scalars is $SO(N)$ invariant.

Throughout this paper we will adopt the convention $8\pi G = 1$.

2 Background Equations and Large N Limit

In this section we review the evolution of homogeneous and isotropic configurations of N scalar fields coupled to Einstein gravity.

For N scalar fields with the Lagrangian density

$$\mathcal{L} = \sum_I \frac{1}{2} g^{\mu\nu} \partial_\mu \phi_I \partial_\nu \phi_I - V(\phi) \quad (1)$$

in a spatially flat Robertson-Walker Universe with the metric

$$ds^2 = dt^2 - a^2(t) dx^2 = a^2(d\eta^2 - dx^2), \quad (2)$$

(the variable t denotes physical time whereas η is conformal time) the equations of motion in the homogeneous background are (with an over-dot denoting the derivative with respect to t)

$$\rho = K + V = 3H^2, \quad (3)$$

$$p = K - V = -2\dot{H} - 3H^2, \quad (4)$$

$$\ddot{\phi}_I + 3H\dot{\phi}_I + V_I = 0. \quad (5)$$

Here, $H \equiv \frac{\dot{a}}{a}$ is the Hubble rate, $K \equiv \sum_I \frac{1}{2} \dot{\phi}_I^2$ and V are the kinetic and potential energy densities, respectively, and $V_I \equiv \frac{\partial V}{\partial \phi_I}$.

From (3) and (4), the time evolution of K and V is completely known if the scale factor $a(t)$ is given

$$K = -\dot{H}, \quad (6)$$

$$V = \dot{H} + 3H^2. \quad (7)$$

As is well-known, not all of the equations (3), (4), and (5) are independent. Indeed, by multiplying (5) by $\dot{\phi}_I$ and summing over I we see that eq. (4), the spatial part of the Einstein equation, can be derived upon using (3).

Conversely, H and \dot{H} are determined by K and V via (6) and (7). In particular, the ratio \dot{H}/H^2 is determined by the ratio K/V . We have smaller \dot{H}/H^2 for smaller K/V . In the inflationary scenario, \dot{H}/H^2 has to be small in order to have a scale invariant spectrum of density perturbation, implying that the potential energy must dominate over the kinetic energy. Below, we will see that potential energy domination arises naturally in the large N limit, as was already realized in [15] (see also [16]).

For a system of N scalar fields coupled to gravity, the kinetic term is proportional to N . A generic potential term of the form $\sum_{I_1 \dots I_n} \lambda_{I_1 \dots I_n} \phi_{I_1} \cdots \phi_{I_n}$ is of order N^n because

it involves n consecutive sums over N fields. For $n > 1$, which holds unless the theory is free, the potential energy dominates over the kinetic energy. Inflation is thus a natural consequence of the large N limit.

The argument above is based on the assumption that the field values ϕ_I and the coupling constants $\lambda_{I_1 \dots I_n}$ do not scale with N . The underlying philosophy is that, in a fundamental theory, all coupling constants are expected to be of order 1 in fundamental units. In string theory, for instance, the couplings among the fields are determined by the string coupling g_s , which is set by the expectation value of the dilaton field. If we include the dilaton as one of the scalar fields, then the natural assumption is that all couplings are of order 1. Note that one aspect of our assumption says that the initial values for all fields should be of order 1. This assumption is related to the types of arguments used to motivate the initial conditions for chaotic inflation [17] (in models of chaotic inflation, the stages of inflation important for comparison with observations - the last 60 e-foldings of inflation - take place when the field values are of the order 1 in Planck units).

Notice that this large N limit is different from that of 't Hooft, where the coupling constants are chosen to scale with N in a particular way such that quantum effects have a well defined limit when the classical background is set to zero. For the ϕ^4 theory, for example, the coupling constant λ scales like $1/N$. The target problem of 't Hooft's large N limit is very different from our consideration of the dynamics of classical background fields.

Having a large number of fields means that, in general, the computations which have to be done in cosmology are more complicated. For instance, we will see below that, to calculate the metric perturbations, we need to solve N coupled differential equations. We will consider two circumstances in which the system of a large number of field fluctuations can be solved in terms of only two fields.

3 Adiabatic and Entropy Perturbations

In this section we give the general formulation for the generation and evolution of perturbations for models with multiple inflatons. Although equivalent formulae were given before in [9, 10, 11], here we present them for completeness and also take this opportunity to introduce the variables κ and μ that will be useful later.

3.1 Adiabatic Perturbation

Fluctuations of the scalar fields $\delta\phi_I$ induce fluctuations of the metric. In the longitudinal gauge (see e.g. [18] for a survey of the theory of cosmological fluctuations), the metric describing scalar metric degrees of freedom is of the form

$$ds^2 = (1 - 2\Phi)dt^2 - a^2(t)(1 + 2\Psi)dx^2, \quad (8)$$

where the two functions Φ and Ψ representing the fluctuations are functions of space and time. For matter consisting of N scalar fields, there are to linear order no non-vanishing off-diagonal space-space components of the energy-momentum tensor. Hence, it follows from the off-diagonal space-space components of the Einstein equations that $\Phi = \Psi$. Vector metric perturbations decay in an expanding Universe and will thus not be considered here. Tensor metric perturbations (gravitational waves) do not couple to matter and their evolution will hence be as in single-field inflation models.

The Einstein equations and the equations of motion for the scalar fields at lowest order are given by (3) and (5). The independent parts of the linear fluctuation equations take the form (see e.g. [10])

$$-3H\dot{\Phi} - \frac{k^2}{a^2}\Phi = \sum_I \frac{1}{2}(\dot{\phi}_I\delta\dot{\phi}_I + V_I\delta\phi_I) + V(\phi)\Phi, \quad (9)$$

$$\dot{\Phi} + H\Phi = \sum_I \frac{1}{2}\dot{\phi}_I\delta\phi_I, \quad (10)$$

$$\delta\ddot{\phi}_I + 3H\delta\dot{\phi}_I + \sum_J \left(\frac{k^2}{a^2}\delta_{IJ} + V_{IJ} \right) \delta\phi_J = 4\dot{\Phi}\dot{\phi}_I - 2V_I\Phi. \quad (11)$$

In linear perturbation theory, each Fourier mode evolves independently. Hence, it is easier to solve the equations in Fourier space (i.e. after having performed a Fourier transformation from comoving spatial coordinates x to comoving wave numbers k). Note that the variables Φ and ϕ_I have a k -dependence which is not manifest in our notation.

The Sasaki-Mukhanov [19] variables for the N scalar fields are

$$Q_I \equiv \delta\phi_I + \frac{\dot{\phi}_I}{H}\Phi. \quad (12)$$

In terms of these variables, the above equations become (see e.g. [10])

$$-\frac{k^2}{a^2}\Phi - (3H^2 + \dot{H}) \left\{ \left(\frac{\Phi}{H} \right)^\cdot + \Phi \right\} = \sum_I \frac{1}{2}(\dot{\phi}_I\dot{Q}_I + V_I Q_I), \quad (13)$$

$$H \left\{ \left(\frac{\Phi}{H} \right)^\cdot + \Phi \right\} = \sum_I \frac{1}{2}\dot{\phi}_I Q_I, \quad (14)$$

$$\ddot{Q}_I + 3H\dot{Q}_I + \frac{k^2}{a^2}Q_I + \sum_J M_{IJ}Q_J = 0. \quad (15)$$

Here

$$\begin{aligned}
M_{IJ} &= V_{IJ} - \frac{1}{a^3} \left(\frac{a^3}{H} \dot{\phi}_I \dot{\phi}_J \right) \cdot \\
&= V_{IJ} + \frac{1}{H} \left[\left(\frac{\dot{H} + 3H^2}{H} \right) \dot{\phi}_I \dot{\phi}_J + V_I \dot{\phi}_J + V_J \dot{\phi}_I \right].
\end{aligned} \tag{16}$$

The comoving curvature perturbation is defined by

$$\zeta \equiv -\frac{H^2}{\dot{H}} \left\{ \left(\frac{\Phi}{H} \right) \cdot + \Phi \right\}. \tag{17}$$

As shown in [20], in the multi-field case ζ becomes

$$\zeta = \frac{H}{2} \frac{\sum_I \dot{\phi}_I Q_I}{K}. \tag{18}$$

The variable ζ is related to the intrinsic three-curvature on the constant energy density surface Σ via

$${}^{(3)}R = \frac{-4k^2}{a^2} \Psi_\Sigma, \tag{19}$$

where $\Psi_\Sigma = \zeta - k^2 \Psi / (3a^2 \dot{H})$. We can use either ζ or equivalently Φ as a measure of the curvature perturbation, which represents the adiabatic mode of the metric perturbations.

We now introduce the two gauge-invariant variables

$$\kappa = \sum_I \dot{\phi}_I Q_I, \quad \mu = \sum_I V_I Q_I. \tag{20}$$

The curvature fluctuation can be written in terms of these two variables as

$$\zeta = -\frac{H\kappa}{2\dot{H}}, \quad \Phi = -\frac{a^2}{2k^2} \left\{ \dot{\kappa} + \left(6H + \frac{\dot{H}}{H} \right) \kappa + 2\mu \right\}. \tag{21}$$

Conversely, we can also express κ and μ in terms of Φ :

$$\kappa = 2H \left\{ \left(\frac{\Phi}{H} \right) \cdot + \Phi \right\}, \tag{22a}$$

$$\mu = -\frac{a^2}{k^2} \Phi - (6H^2 - 4\dot{H} - \frac{\ddot{H}}{H}) \Phi - 7H\dot{\Phi} - \ddot{\Phi}. \tag{22b}$$

In the following subsection we will see that also the entropy fluctuation can be written in terms of the two variables μ and κ .

3.2 Entropy Perturbation

A dimensionless gauge-invariant definition of the total entropy perturbation is (see e.g. [21])

$$\mathcal{S} = H \left(\frac{\delta p}{\dot{p}} - \frac{\delta \rho}{\dot{\rho}} \right). \quad (23)$$

For N scalar fields, this yields

$$\mathcal{S} = \frac{2(\dot{V} + 3H \sum_I \dot{\phi}_I^2) \delta V + 2\dot{V} \sum_I \dot{\phi}_I (\delta \dot{\phi}_I - \dot{\phi}_I \Phi)}{3(2\dot{V} + 3H \sum_I \dot{\phi}_I^2) \sum_J \dot{\phi}_J^2}. \quad (24)$$

In terms of κ and μ , we have [11]

$$\mathcal{S} = - \frac{2(\ddot{H} + 3H\dot{H})\mu + (\ddot{H} + 6H\dot{H}) \left[\dot{\kappa} + \frac{\dot{H} + 3H^2}{H} \kappa \right]}{6\dot{H}(\ddot{H} + 3H\dot{H})}. \quad (25)$$

We see that both adiabatic and entropy perturbations can be expressed in terms of the two variables κ and μ .

On the other hand, if we express the entropy perturbation in terms of Φ , we have

$$\mathcal{S} = \frac{k^2}{3a^2\dot{H}}\Phi - \frac{H^2}{3H\dot{H} + \ddot{H}} \left\{ \left(\frac{\Phi}{H} \right)' + \Phi \right\}. \quad (26)$$

In the long wave length limit, it turns out that [22, 10, 21, 11]

$$\dot{\zeta} \simeq \frac{\dot{p}}{\rho + p} \mathcal{S}. \quad (27)$$

Assuming the slow roll condition ($\dot{H} \ll H^2$), the above becomes

$$\dot{\zeta} \simeq 3H \left(\frac{\dot{p}}{\dot{\rho}} - \frac{\delta p}{\delta \rho} \right) \zeta. \quad (28)$$

In the case of a single inflaton, \mathcal{S} vanishes and curvature perturbations ζ are frozen outside the Hubble radius.

In the case of multiple fields, the entropy perturbation is in general not zero, and it can be important in determining the magnitude of adiabatic perturbation. For example, as shown in [10] and [23], entropy fluctuations can seed exponential growth of super-Hubble curvature fluctuations during inflationary reheating. In our case, when the inflatons are slowly rolling down the potential hill, we have $\dot{p} > 0$ and $(\rho + p) > 0$. Since $\dot{H} < 0$, Equations (26) and (27) imply that the amplitude of super-Hubble-scale curvature perturbations will increase due to the coupling to the isocurvature mode.

3.3 Generation of Quantum Fluctuations

Since the inflationary expansion redshifts all classical perturbations pre-existing before inflation, it leaves behind a quantum vacuum. In the context of inflationary cosmology, quantum vacuum fluctuations are believed to be the origin of the fluctuations we observe today. In quantizing the cosmological perturbations, it is important to identify the variables in terms of which the action for fluctuations takes on canonical form, i.e. in terms of which the fluctuation fields have the usual kinetic term. As reviewed in [18], the canonical variables v_I are related to the Q_I variables via $v_I = a(t)Q_I$. Hence, if we perform standard canonical quantization, and expand the canonical fields into creation and annihilation operators, then the expansion of the fields Q_I in terms of these operators takes the form (at the initial time t_0)

$$Q_I(t_0) = \frac{1}{\sqrt{2ka(t_0)}} \left(\hat{a}_I(t_0) + \hat{a}_I^\dagger(t_0) \right), \quad (29)$$

$$P_I(t_0) = -ia(t_0)\sqrt{\frac{k}{2}} \left(\hat{a}_I(t_0) - \hat{a}_I^\dagger(t_0) \right), \quad (30)$$

where the P_I are the momenta conjugate to Q_I

$$P_I(t) = a^2(t)\dot{Q}_I(t) + a^2(t)H Q_I(t). \quad (31)$$

Since the evolution of Q_I (15) satisfies linear differential equations, Q_I and P_I at a later time t must depend on the initial data linearly,

$$Q_I(t) = \frac{1}{\sqrt{2ka(t)}} \sum_J \left(q_{IJ}(t) \hat{a}_J(t_0) + \bar{q}_{IJ}(t) \hat{a}_J^\dagger(t_0) \right), \quad (32)$$

$$P_I(t) = -ia(t)\sqrt{\frac{k}{2}} \sum_J \left(p_{IJ}(t) \hat{a}_J(t_0) - \bar{p}_{IJ}(t) \hat{a}_J^\dagger(t_0) \right), \quad (33)$$

where the functions $q_{IJ}(t)/a(t)$ satisfy (15) for all J and

$$p_{IJ} = i \frac{\dot{q}_{IJ}}{k}. \quad (34)$$

Due to (29), the initial conditions are

$$q_{IJ}(t_0) = p_{IJ}(t_0) = \delta_{IJ}. \quad (35)$$

To solve the differential equation for $q_{IJk}(t)$, we need the initial values for both $q_{IJk}(t_0)$ and $\dot{q}_{IJk}(t_0)$. The latter are (according to (34)) given by

$$\dot{q}_{IJ}(t_0) = -ik\delta_{IJ}. \quad (36)$$

The two point correlation function at time t for the vacuum state defined at t_0 , denoted $|0\rangle_0$, is

$${}_0\langle 0|Q_I(t)Q_J(t)|0\rangle_0 = \frac{1}{2ka^2(t)} \left| \sum_K q_{IK}(t)q_{KJ}^\dagger(t) \right| \delta^{(3)}(k-k'). \quad (37)$$

(On the left hand side, we omitted the k -dependence of Q_I and Q_J , which should be denoted Q_{Ik} and $Q_{Jk'}$.)

As we have seen in the previous section, the quantities ζ , Φ and \mathcal{S} , which we are interested in, can be expressed in terms of κ and μ (20). Consequently, their correlation functions can also be expressed in terms of those of κ , μ and their derivatives.

In general, this calculation involves solving the equations of motion for each of the N fields Q_I . However, we will see in the following sections that, in certain cases, the evolution equations of κ and μ form a set of coupled differential equation by themselves. As a result, they can be completely determined without having to know the evolution of individual components Q_I .

4 Slow Roll

The “slow roll” conditions are usually defined as

$$|\dot{H}| \ll H^2, \quad |\ddot{H}| \ll |\dot{H}H|. \quad (38)$$

It then follows from (6), (7) that

$$V \gg K, \quad \dot{V} \gg \dot{K}. \quad (39)$$

For the case of a single inflaton, the slow roll conditions imply that $|\frac{dV}{d\phi}| \ll V$, since

$$\left| \frac{dV}{d\phi} \right| = \frac{|\dot{V}|}{|\dot{\phi}|} \simeq \frac{6|\dot{H}|H}{\sqrt{2K}} \simeq 3\sqrt{2|\dot{H}|H} \ll \sqrt{2}V. \quad (40)$$

Similarly one can easily show that $|\frac{d^2V}{d\phi^2}| \ll V$. In this case, the slow roll conditions also imply that the second time derivative of the scalar field is negligible. We will now show that this last conclusion does not hold if there are many inflaton fields (in which case the use of the term “slow rolling” is in fact somewhat misleading).

In the case of a large number of inflatons, we still need the vector $\dot{\phi}_I$ to be small, so that $K \ll V$. However, we can arrange $\ddot{\phi}_I$, viewed as an N -vector, to be roughly perpendicular to $\dot{\phi}_I$ such that $\ddot{H} = -\dot{\phi}_I \ddot{\phi}_I$ is nearly zero. This can occur even if $\ddot{\phi}_I$ is not

small. (Note that $\dot{\phi}_I$ can not be directly related to V_I if $\ddot{\phi}_I$ is not small.) In addition, since $\dot{\phi}_I$ may be vanishingly small at a certain moment, $V_I = \dot{V}/\dot{\phi}_I$ can be very large compared to V . Therefore, it may not be a good approximation to ignore M_{IJ} (16) even if the slow roll conditions are satisfied, and the calculation for fluctuations will in general be much more complicated than in the case of a single inflaton.

In the following we consider two situations in which the theory of perturbations is greatly simplified, and in which we will not have to solve N coupled equations for each Q_I , but only have two coupled equations for κ and μ .

5 Smooth Potential

Following the previous section, we see that the “slow roll” conditions are not enough to guarantee the scale invariance of the spectrum of fluctuations if there are more than one inflatons, the reason being that the M_{IJ} terms in Equation (16) are not in general negligible. In this section we consider the case when the potential is very flat and show that adiabatic and entropy perturbations can be determined by the two variables κ and μ alone. The scale invariance of the spectrum will result if we assume that the slow roll conditions hold.

Suppose that the potential is very flat, more precisely,

$$V_{IJ} \ll \dot{\phi}_I \dot{\phi}_J, \quad (41)$$

$$V_{IJK} \ll \dot{\phi}_I \dot{\phi}_J \dot{\phi}_K \quad (42)$$

in Planck units. It follows from (41) that

$$M_{IJ} \simeq \frac{1}{H} \left(\frac{\dot{H} + 3H^2}{H} \dot{\phi}_I \dot{\phi}_J + V_I \dot{\phi}_J + V_J \dot{\phi}_I \right). \quad (43)$$

It is straightforward to derive the equations of motion for κ and μ

$$\ddot{\kappa} + 9H\dot{\kappa} + \left(\frac{k^2}{a^2} + 18H^2 + 3\dot{H} - 2\frac{\dot{H}^2}{H^2} + \frac{\ddot{H}}{H} \right) \kappa + 2\dot{\mu} + \frac{6H^2 - 2\dot{H}}{H} \mu \simeq 0, \quad (44)$$

$$\ddot{\mu} + 3H\dot{\mu} + \left(\frac{k^2}{a^2} + 6\dot{H} + \frac{\ddot{H}}{H} \right) \mu - \left(\frac{H^{(3)}}{H} + 6\ddot{H} - \frac{\dot{H}\ddot{H}}{H^2} \right) \kappa \simeq 0. \quad (45)$$

In deriving the above we used (7) and

$$V_I V_I \simeq -(\ddot{V} + 3H\dot{V}), \quad (46)$$

which in turn can be derived from Equation (5) by multiplying it with V_I and making use of Equation (41).

If we also assume the slow roll conditions (38), the equations are simplified to

$$\hat{\kappa}'' + (k^2 - 2\mathcal{H}^2)\hat{\kappa} + 2a^4(\hat{\mu}' + 2\mathcal{H}\hat{\mu}) = 0, \quad (47)$$

$$\hat{\mu}'' + (k^2 - 2\mathcal{H}^2)\hat{\mu} = 0, \quad (48)$$

where

$$\hat{\kappa} = a^4 \kappa, \quad \hat{\mu} = a\mu, \quad \mathcal{H} = \frac{a'}{a}, \quad (49)$$

and the primes refer to derivatives with respect to conformal time

$$d\eta = \frac{dt}{a}. \quad (50)$$

The slow roll conditions imply

$$\mathcal{H}' \simeq \mathcal{H}^2 \simeq \frac{a''}{2a}. \quad (51)$$

Equation (48) is the key equation needed to demonstrate that a scale-invariant spectrum results, as we now show. A fluctuation mode with wave number k crosses the Hubble radius at a time η_k when $k \simeq \mathcal{H}$, or equivalently when

$$H = \frac{k}{a(\eta_k)}. \quad (52)$$

For $\eta \ll \eta_k$, so that $k \gg \mathcal{H}$, we see from Equations (47) and (48) that the fluctuation is in the oscillatory phase. The amplitude of the canonical fluctuation variable remains the same. Hence, the amplitude of the fluctuations when they cross the Hubble radius is the vacuum amplitude, and as in the single inflaton case this implies that the spectrum will be scale invariant.

To see this explicitly, consider that when $\eta \gg \eta_k$, so that $k \ll \mathcal{H}$, the fluctuation $\hat{\kappa}$ grows like a . The approximate solution of the equations is

$$\hat{\mu} = \hat{\mu}_0 \frac{a(\eta)}{a(\eta_k)}, \quad \hat{\kappa} = \hat{\kappa}_0 \frac{a(\eta)}{a(\eta_k)} - \frac{\hat{\mu}_0}{3a(\eta_k)} \frac{a^5(\eta)}{\mathcal{H}}, \quad (53)$$

where $\hat{\kappa}_0$ and $\hat{\mu}_0$ are the initial values of $\hat{\kappa}$ and $\hat{\mu}$ prescribed at a moment $\eta_0 < \eta_k$. To find the dependence on k of the two point correlation function of ζ (or Φ), we use (21) and (37). Since the dominant mode of $\hat{\kappa}$ scales as $a(t)^4$ (from Equation (53), κ is independent of time, and hence

$${}_0\langle 0|\zeta_k\zeta_k|0\rangle_0 \propto \frac{1}{2ka^2(\eta_k)} \propto \frac{1}{k^3}, \quad (54)$$

where we employed (52). This corresponds to a scale invariant spectrum.

Let us remark here that a scale invariant spectrum at the Hubble radius during inflation does not always imply the same for the observed spectrum of density perturbations because, as we have mentioned earlier (around (27)), the entropy perturbation may lead to a growth of the adiabatic perturbation in time. In the present case, however, we can solve for both the adiabatic and the entropy modes, i.e. for both κ and μ (both are constant on super-Hubble scales), and the joint analysis of the equations lead to Φ having an approximately scale invariant spectrum.

6 $SO(N)$ Symmetry

In this section we study another situation where we can completely determine the evolution of perturbations by solving for only two independent variables, without having to know each of the N variables Q_I . Consider the potential term

$$V = V(B), \quad (55)$$

where

$$B \equiv \sum_I \phi_I^2, \quad (56)$$

so that the system of inflatons has $SO(N)$ symmetry. By multiplying Equation (5) by ϕ_I and summing over I , we have

$$\ddot{B} + 3H\dot{B} + 4B\frac{dV(B)}{dB} + 4\dot{H} = 0. \quad (57)$$

Together with Equation (7), B and H form a set of coupled differential equations. Given $B(t_0)$, $\dot{B}(t_0)$, and $H(t_0)$, we can determine B and H uniquely. We can then solve for K from Equation (3). Hence, these equations completely specify the dynamics of the background. Since the dependence of B on H is via the functional form of V , the result is independent of the value of N .

The one special case is $N = 1$. In this case, we have

$$\dot{B} = 2\phi\dot{\phi}. \quad (58)$$

Making use of Equation (6), we obtain another equation for B and H , namely

$$\dot{B}^2 = -8B\dot{H}, \quad (59)$$

which implies Equation (57). In this case, we only need $B(t_0)$ and $H(t_0)$ to specify the dynamics and the phase space is reduced by one dimension. Equivalently, we can

use Equations (7) and (57) to determine B and H like in the case of multiple inflatons except that $\dot{B}(t_0)$ is now fixed by $B(t_0)$ and $H(t_0)$ through Equations (7) and (59).

We emphasize that although the theory itself has an $SO(N)$ symmetry, the classical background is not restricted to be $SO(N)$ invariant. Remarkably, as we will see below, we can still determine the metric perturbations by dealing only with two variables.

Let us now return to the equations for the perturbations for generic N . Equation (15) implies that

$$\begin{aligned} \ddot{\kappa} + 9H\dot{\kappa} + \left(\frac{k^2}{a^2} + 3\dot{H} + 18H^2 - 2\frac{\dot{H}^2}{H^2} + \frac{\ddot{H}}{H} \right) \kappa \\ + 2\dot{\mu} + \left(6H - 2\frac{\dot{H}}{H} \right) \mu = 0, \end{aligned} \quad (60)$$

$$\begin{aligned} \ddot{\mu} + (3H - 2u)\dot{\mu} + \left(\frac{k^2}{a^2} + 6\dot{H} + \frac{\ddot{H}}{H} + u^2 - 3Hu - \dot{u} + 4B\frac{d^2V}{dB^2} \right) \mu \\ - 4\frac{dV}{dB}\dot{\kappa} - \left\{ 12H\frac{dV}{dB} - \frac{4}{H}B\left(\frac{dV}{dB}\right)^2 - \frac{1}{H^2}(\dot{H} + 3H^2)(\ddot{H} + 6H\dot{H}) \right\} \kappa = 0. \end{aligned} \quad (61)$$

Here,

$$u = \frac{d}{dt} \log\left(\frac{dV}{dB}\right) = \frac{\frac{d^2V}{dB^2}(\ddot{H} + 6H\dot{H})}{\left(\frac{dV}{dB}\right)^2}, \quad (62)$$

where we have used $\frac{dV}{dB}\dot{B} = \dot{V}$.

If the inverse function $B = B(V)$ exists, then B , V , $\frac{dV}{dB}$ and $\frac{d^2V}{dB^2}$ can be viewed as given functions of time, and thus both of the above differential equations can be used to solve for κ and μ . In turn, these results can then be used to determine ζ , \mathcal{S} and Φ . Our goal is to study under which conditions the resulting spectrum of fluctuations will be scale-invariant, starting with an initial vacuum state on sub-Hubble scales at the initial time. One obtains a scale-invariant spectrum if the Fourier modes of the canonically normalized fields representing the fluctuations undergo oscillations with constant amplitude while on sub-Hubble scales, and are then squeezed with amplitude proportional to the scale factor $a(t)$ on super-Hubble scales (see e.g. [18, 24] for comprehensive reviews). For the fields Q_I these requirements imply that we want Q_I to undergo damped oscillations on sub-Hubble scales (with the amplitude decreasing as $a^{-1}(t)$, and to be squeezed as in the case of the previous section on super-Hubble scales).

Assuming the slow roll conditions (38), we see u and \dot{u} are approximately zero and the Equations (60), (61) can be simplified. In terms of the variables (49) and conformal time, these equations are

$$\hat{\kappa}'' + (k^2 - 2\mathcal{H}^2) \hat{\kappa} + 2a^4(\hat{\mu}' + 2\mathcal{H}\hat{\mu}) = 0, \quad (63)$$

$$\hat{\mu}'' + \left(k^2 - 2\mathcal{H}^2 + 4a^2B\frac{d^2V}{dB^2} \right) \hat{\mu} - 4a^{-2}\frac{dV}{dB}\hat{\kappa}' + \left(\frac{4\mathcal{H}\frac{dV}{dB}}{a^2} + \frac{4B\left(\frac{dV}{dB}\right)^2}{\mathcal{H}} \right) \hat{\kappa} = 0. \quad (64)$$

Notice that the primes mean derivatives with respect to η .

A priori, these coupled equations admit the possibility of having more than the two phases (sub-Hubble and super-Hubble) described above. In the case of a single inflaton, there is a single transition (at Hubble radius crossing) for each fluctuation mode from the oscillatory phase to the frozen phase. In the case of many inflatons, a fluctuation mode may experience more than one transition between oscillatory and frozen phases before the end of inflation. However, for the following explicit examples, we do not find multiple phases. Instead, we find that some fluctuation modes never experience a frozen phase, like what happens for a single scalar field with a very large mass.

6.1 Massive Fields

To begin, we consider the simplest case of free fields with

$$V(B) = \frac{1}{2}m^2 B. \quad (65)$$

In this case $\frac{dV}{dB} = m^2/2$ and $\frac{d^2V}{dB^2} = 0$. The time evolution of B is determined by Equations (7) and (57). Using the definition $\tilde{m} = m/H$, Equations (63) and (64) can be simplified to yield

$$\hat{\kappa}'' + (k^2 - 2\mathcal{H}^2)\hat{\kappa} + 2a^4(\hat{\mu}' + 2\mathcal{H}\hat{\mu}) = 0, \quad (66)$$

$$\hat{\mu}'' + (k^2 - 2\mathcal{H}^2)\hat{\mu} - 2\tilde{m}^2\mathcal{H}^2a^{-4}(\hat{\kappa}' - 4\mathcal{H}\hat{\kappa}) = 0. \quad (67)$$

Eliminating $\hat{\mu}$ from the above two equations, we can obtain a fourth differential equation for $\hat{\kappa}$:

$$\begin{aligned} &\hat{\kappa}'''' - 8\mathcal{H}\hat{\kappa}''' + \{2k^2 + 4(1 + \tilde{m}^2)\mathcal{H}^2\}\hat{\kappa}'' - \{8k^2 + 8(-1 + 2\tilde{m}^2)\mathcal{H}^2\}\mathcal{H}\hat{\kappa}' \\ &+ \{k^4 + 4k^2\mathcal{H}^2 + 8(1 - 2\tilde{m}^2)\mathcal{H}^4\}\hat{\kappa} = 0. \end{aligned} \quad (68)$$

Once we solve the above equation for $\hat{\kappa}$, we can find $\hat{\mu}$ by using Equation (66).

To get a rough idea of the solutions to these equations, we consider the following limits.

- (1) $\mathcal{H} \ll k$ (sub-Hubble), with \tilde{m} fixed.

Equation (68) can be solved order by order in \mathcal{H}/k . For more detailed analysis, we consider the case of exponential inflation which is consistent with the slow roll condition. In this case

$$a = -1/(H\eta), \quad (69)$$

and H is a constant.

(a) For $\tilde{m} \ll 1$, it reduces to the case of a smooth potential. We find

$$\hat{\kappa} \simeq c_1 \eta^{-3} \cos(k\eta + \theta_1) + c_2 \cos(k\eta + \theta_2); \quad (70)$$

$$\hat{\mu} \simeq 3H^4 c_1 \cos(k\eta + \theta_1). \quad (71)$$

Hence, both $\hat{\kappa}$ and $\hat{\mu}$ are in the oscillatory phase. The amplitude of the oscillation of μ is decreasing as $a^{-1}(t)$ as we need it to in order to obtain a scale invariant spectrum. (The relation between μ and $\hat{\mu}$ is given in (49).)

(b) For $\tilde{m} \gg 1$, the fluctuations are in the oscillatory phase with a different amplitude evolution

$$\hat{\kappa} \propto \eta^{-3/2} \cos(k\eta + \theta_1) \cos(\tilde{m} \log \eta + \theta_2); \quad (72)$$

$$\hat{\mu} \propto \eta^{3/2} \cos(k\eta + \theta_1) \cos(\tilde{m} \log \eta + \theta_2). \quad (73)$$

(2) $\mathcal{H} \gg k$ (super-Hubble), with \tilde{m} fixed.

Under this condition, and with (69), Equation (68) becomes

$$\hat{\kappa}'''' + \frac{8}{\eta} \hat{\kappa}''' + \frac{4(1 + \tilde{m}^2)}{\eta^2} \hat{\kappa}'' - \frac{8(1 - 2\tilde{m}^2)}{\eta^3} \hat{\kappa}' + \frac{8(1 - 2\tilde{m}^2)}{\eta^4} \hat{\kappa} = 0. \quad (74)$$

Similarly, we can find the solutions of $\hat{\kappa}$ and $\hat{\mu}$:

$$\hat{\kappa} = c_2 \eta^{-4} + c_3 \eta^{\frac{1+\sqrt{9-16\tilde{m}^2}}{2}} + c_4 \eta^{\frac{1-\sqrt{9-16\tilde{m}^2}}{2}}, \quad (75)$$

$$\begin{aligned} \hat{\mu} = H^4 \left\{ c_1 \eta^2 + 3c_2 \eta^{-1} + \left(\frac{3 - \sqrt{9 - 16\tilde{m}^2}}{4} \right) c_3 \eta^{\frac{7+\sqrt{9-16\tilde{m}^2}}{2}} \right. \\ \left. + \left(\frac{3 + \sqrt{9 - 16\tilde{m}^2}}{4} \right) c_4 \eta^{\frac{7-\sqrt{9-16\tilde{m}^2}}{2}} \right\}. \end{aligned} \quad (76)$$

(a) For $\tilde{m} \ll 1$, the evolution equations reduce to the slow roll scenario with a shallow potential in Sec. 5. κ and μ freeze out and the wave functions are squeezed as in single-field inflation. Hence, the resulting spectrum is scale invariant.

(b) For $\tilde{m} \gg 1$, the large mass induces an oscillation of $\hat{\kappa}$ and $\hat{\mu}$ even on these super-Hubble scales:

$$\hat{\kappa} \propto \eta^{1/2} \cos(2\tilde{m} \ln \eta); \quad (77)$$

$$\hat{\mu} \propto \eta^{7/2} \sin(2\tilde{m} \ln \eta). \quad (78)$$

To summarize, in an accelerating Universe, a fluctuation mode with given k starts in the oscillatory phase (quantum vacuum oscillations) with $k \gg \mathcal{H}$ (Case (1)). If $\tilde{m} \ll 1$, then when the mode crosses the Hubble radius it freezes out and its amplitude grows. Thus, if $\tilde{m} \ll 1$ at the time of Hubble radius crossing, the qualitative picture of the evolution of fluctuations is the same as in the case of a single inflaton.

On the other hand, if $\tilde{m} \gg 1$ at Hubble radius crossing, κ keeps oscillating even on super-Hubble scales. Hence, the generation of classical fluctuations is suppressed.

Except in the case of exponential inflation, generically each fluctuation mode crosses the Hubble radius with a different Hubble parameter H and thus a different value of \tilde{m} . For power law inflation, i.e. $a(t) \propto t^n$ with $n > 1$, H decreases with time, and so $m \ll H$ for lower frequency modes which cross the Hubble radius earlier, and $m \gg H$ for higher frequency modes. Consequently, the high frequency part of the spectrum is suppressed. This effect has the wrong sign to explain the suppression of the spectrum at large wavelengths (observed in the recent CMB data [25]) compared to what single field inflation models predict.

6.2 ϕ^4 Theory

Finally we consider the ϕ^4 theory, which is the interesting case in the context of the large N limit mentioned in Section 2.

For simplicity, we ignore the mass term in the potential and write

$$V(B) = \frac{\lambda}{4} B^2. \quad (79)$$

From (7) we find $B \simeq \sqrt{\frac{12}{\lambda}} H$. Making use of the definition $\tilde{\lambda} = 27\lambda/H^2$, it follows from (63), (64) that

$$\hat{\kappa}'' + (k^2 - 2\mathcal{H}^2) \hat{\kappa} + a^4 \{2\hat{\mu}' + 4\mathcal{H}\hat{\mu}\} = 0, \quad (80)$$

$$\hat{\mu}'' + \left\{ k^2 + (-2 + \frac{4}{3}\sqrt{\tilde{\lambda}})\mathcal{H}^2 \right\} \hat{\mu} - \frac{4}{3}\sqrt{\tilde{\lambda}}\mathcal{H}^2 a^{-4} \{ \hat{\kappa}' - 7\mathcal{H}\hat{\kappa} \} = 0. \quad (81)$$

As in the previous subsection, we can combine these two equations to obtain a fourth order differential equation for $\hat{\kappa}$:

$$\begin{aligned} (k^2 + \frac{4}{3}\sqrt{\tilde{\lambda}}\mathcal{H}^2)\hat{\kappa}'''' &- 8(k^2 + \frac{1}{3}\sqrt{\tilde{\lambda}}\mathcal{H}^2)\mathcal{H}\hat{\kappa}''' + \left\{ 2k^4 + 4k^2(1 + \frac{5}{3}\sqrt{\tilde{\lambda}})\mathcal{H}^2 + \frac{16}{3}(4\sqrt{\tilde{\lambda}} + \tilde{\lambda})\mathcal{H}^4 \right\} \hat{\kappa}'' \\ &- \left\{ 8k^4 + 8k^2(-1 + 4\sqrt{\tilde{\lambda}})\mathcal{H}^2 + 16(-\sqrt{\tilde{\lambda}} + 2\tilde{\lambda})\mathcal{H}^4 \right\} \mathcal{H}\hat{\kappa}' \\ &+ \left\{ k^6 + 4k^4(1 + \frac{2}{3}\sqrt{\tilde{\lambda}})\mathcal{H}^2 + 8k^2(1 + \frac{2}{9}\tilde{\lambda})\mathcal{H}^4 + \frac{32}{3}(-\sqrt{\tilde{\lambda}} + 2\tilde{\lambda})\mathcal{H}^6 \right\} \hat{\kappa} = 0. \end{aligned} \quad (82)$$

The discussion of the growth of adiabatic perturbations parallels that of the previous subsection. The fluctuation starts in a phase of vacuum oscillations and then, for a certain region of wavelengths, freezes out at Hubble radius crossing. The other wavelength modes remain in the oscillatory phase even after crossing the Hubble radius. Let us describe each phase separately.

(1) $k \gg \mathcal{H}$, with $\tilde{\lambda}$ fixed.

Assuming $\tilde{\lambda} \ll 1$, this is the short wavelength phase of vacuum oscillations in which $\hat{\kappa}$ and $\hat{\mu}$ evolve in exactly the same as given in the previous subsection (Equations (70) and (71)). This is far from surprising since only terms with leading k dependence survive in this limit. If $\tilde{\lambda} \gg 1$, the result is the same as (72) and (73) with \tilde{m}^2 replaced by $\frac{2}{3}\sqrt{\tilde{\lambda}}$.

(2) $k \ll \mathcal{H}$, with $\tilde{\lambda}$ fixed.

To be specific, we again consider the case of exponential inflation. In this region (long wavelength region), Equation (82) becomes

$$\hat{\kappa}'''' + \frac{10}{\eta}\hat{\kappa}''' + \frac{4(4 + \sqrt{\tilde{\lambda}})}{\eta^2}\hat{\kappa}'' + \frac{12(-1 + 2\sqrt{\tilde{\lambda}})}{\eta^3}\hat{\kappa}' + \frac{8(-1 + 2\sqrt{\tilde{\lambda}})}{\eta^4}\hat{\kappa} = 0. \quad (83)$$

As in the previous subsection, we can find the solutions of $\hat{\kappa}$ and $\hat{\mu}$:

$$\hat{\kappa} = c_1\eta^{-1} + c_2\eta^{-4} + c_3\eta^{\frac{1+\sqrt{9-16\sqrt{\tilde{\lambda}}}}{2}} + c_4\eta^{\frac{1-\sqrt{9-16\sqrt{\tilde{\lambda}}}}{2}}, \quad (84)$$

$$\begin{aligned} \hat{\mu} = H^4 \left\{ 6c_1\eta^2 + 3c_2\eta^{-1} + \left(\frac{3 - \sqrt{9 - 16\sqrt{\tilde{\lambda}}}}{4} \right) c_3\eta^{\frac{7+\sqrt{9-16\sqrt{\tilde{\lambda}}}}{2}} \right. \\ \left. + \left(\frac{3 + \sqrt{9 - 16\sqrt{\tilde{\lambda}}}}{4} \right) c_4\eta^{\frac{7-\sqrt{9-16\sqrt{\tilde{\lambda}}}}{2}} \right\}, \quad (85) \end{aligned}$$

Again there are two cases:

- (a) $\tilde{\lambda} \ll 1$. In this case, the fluctuations are frozen and their amplitude grows (with the leading time dependence the same, but the sub-leading terms slightly different from what occurs in the slow roll scenario for a flat potential in Sec. 5). The spectrum is still scale invariant.

(b) $\tilde{\lambda} \gg 1$. In this case, the fluctuations are oscillatory even on super-Hubble scales, with

$$\hat{\kappa} \propto \eta^{1/2} \cos(2\tilde{\lambda}^{1/4} \ln \eta); \quad (86)$$

$$\hat{\mu} \propto \eta^{7/2} \sin(2\tilde{\lambda}^{1/4} \ln \eta). \quad (87)$$

This result is qualitatively similar to what was obtained in the previous subsection.

7 Discussion

In this paper we have studied inflationary dynamics of a system of N scalar fields coupled to Einstein gravity as a model for the very early Universe. In the large N limit, the probability for slow roll inflation increases. We showed that if the scalar fields respect $SO(N)$ symmetry, the phase space for the homogeneous, isotropic background is three dimensional (B, \dot{B}, H) , only one dimension more than the case of a single scalar field. Furthermore, for arbitrary N , we showed that only two independent variables (κ, μ) are needed to determine both adiabatic and entropy perturbations, and not N as could be naively expected. In the case of a single inflaton field, only one variable is needed since there is (on super-Hubble scales) only the adiabatic fluctuation mode. This simplification allows us to analyze the fluctuation spectrum and find its deviation from the case of a single inflaton. Note that this model is independent of N in terms of the appropriate variables. It will be interesting to build models of this class to explain experimental data and compare with single-inflaton models.

Note that, as shown in Section 2, initial conditions with field values of order one in Planck units may induce slow roll inflation in the large N limit. It would be interesting to consider the possibility of inflationary models with trivial classical background, and where inflation is induced purely by quantum fluctuations. In this case, a new formulation for the generation of fluctuations has to be constructed (the usual theory requires a non-vanishing classical background matter field). We leave this interesting question for future study (for early work on this issue see [26]).

Upon completion of this work, we were informed of a paper [27] which deals with complementary aspects of large N inflation.

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